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Reaching out through the MOOC Introduction to Calculus

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Abstract

The MOOC Introduction to Calculus was launched in December 2018, in time for students, in transition from school, to use for satisfying mathematics prerequisites for tertiary-level degrees. The MOOC was conceived holistically, contributing towards improving diversity and inclusion, creating opportunities for participants throughout the world, including those from regions suffering from political or economic instability. The MOOC facilitates pathways towards higher education through greater awareness of mathematics and its usefulness. This became acutely relevant for students facing disruptions from COVID-19 and forced into remote learning. Historical narratives permeate the MOOC, providing students with perspectives that they might not see in typical classrooms or textbooks. The MOOC alleviates frustration from students who have difficulty navigating online material or who have had negative experiences in learning mathematics. It uses a mastery model of learning and was designed using principles from the theory of threshold concepts and learning phases in the SOLO taxonomy.

Keywords: *MOOC; calculus; mastery learning; threshold concepts; SOLO taxonomy.*

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1. Introduction

This article provides background and a snapshot of progress of a recent initiative by Sydney University, in launching the Massive Open Online Course (MOOC) *Introduction to Calculus,* in December 2018. This course combines flexible online technology with the common language and culture of mathematics and calculus. Course materials are freely available and designed and developed with the intention of promoting inclusion and bringing people together from diverse backgrounds as part of a global community of learners.

This project gained particular importance at the time of the launch, due to the introduction of mathematics prerequisites at Sydney University for certain courses, in science, business and economics, commencing in 2019 (University of Sydney, 2019). The MOOC was especially aimed at providing opportunities for students from disadvantaged backgrounds, from rural or remote areas, or from schools lacking resources or qualified mathematics teachers, to study at the Sydney University from First Semester 2019. Completion of the MOOC has been accepted as equivalent to satisfying mathematics prerequisites for enrolment. This pathway has been taken successfully now by several hundred students, in preparing for enrolments in First or Second Semester in courses at Sydney University, over the period 2019-2022. Processes have been put in place to continue to provide these opportunities to prospective students in future semesters.

The MOOC has been conceived and designed holistically and has had other significant benefits, in particular,

- to help teachers in the secondary and tertiary sectors upgrade their skills and become better qualified to teach calculus and foundational mathematics, and,
- to provide outreach to people of all ages and backgrounds from all over the world.

The MOOC has been adopted widely, even by participants from regions experiencing political or economic turmoil or upheaval (see Table 1). The only technical requirements for participation are reasonable internet access and use of an electronic device that can view videos, view or download notes in pdf format, interact with online multiple-choice quizzes and make posts on threads in the discussion forums. Even though the MOOC is formally constructed as a 'five-week' course (see below), students may work at their own pace, and pick up from where they left off if there is a delay in their participation for any reason.

The MOOC fits neatly within the transition mode from school to tertiary studies. It also reinforces and aids the preparation of students already in the tertiary system, who need mathematics for their chosen area of study but may find entry difficult and mastery of the methods and techniques problematic. The MOOC creates a flexible and collegial online learning environment, lubricating pathways for students through their individual *liminal*

Table 1. MOOC cumulative active learners. Source: (Coursera Analytics, 1 February 2022).

India	31,394	Portugal	273	Mauritius	46
United States	31,043	Tunisia	248	Mongolia	46
Australia	6,532	Hungary	247	Tanzania	44
Philippines	5,727	Ethiopia	246	Yemen	44
Canada	4,983	Somalia	242	Bhutan	44
Pakistan	3,340	Oman	235	Bahamas	41
United Kingdom	3,309	Finland	223	Maldives	38
Nigeria	3,281	Denmark	203	Macao	38
Bangladesh	3,244	Norway	193	Grenada	36
Turkey	2,762	Uzbekistan	189	Luxembourg	36
Egypt	2,605	Austria	180	Slovenia	34
Brazil	2,585	Czech Republic	175	Bosnia	32
Mexico	2,452	Bolivia	174	Angola	30
China	1,992	Jordan	173	Namibia	30
Germany	1,966	Qatar	157	Iceland	29
Indonesia	1,559	Costa Rica	146	Moldova	27
Hong Kong	1,523	Bulgaria	144	Belize	26
Russian Federation	1,423	Guatemala	142	Cote d'Ivoire	24
Singapore	1,398	Kuwait	134	Lesotho	23
Colombia	1,262	Dominican Republic	127	Brunei	23
Thailand	1,136	Venezuala	124	Malawi	22
France	1,030	Georgia	120	Malta	20
Netherlands	1,005	Cambodia	120	Fiji	19
Italy	941	Armenia	116	Liberia	19
Saudi Arabia	912	Panama	115	Senegal	18
United Arab Emirates	904	Trinidad and Tobago	115	Gambia	18
South Africa	899	Serbia	113	Mozambique	17
Taiwan	845	Zambia	108	Papua New Guinea	16
South Korea	821	Estonia	97	Republic of Congo	14
Japan	790	Uganda	97	Madagascar	13
Spain	760	Jamaica	96	Guam	13
Vietnam	756	Kyrgyzstan	89	Swaziland	13
Nepal	731	Honduras	84	Kosova	13
Malaysia	697	Lithuania	83	Tajikistan	12
Ghana	691	Sudan	82	Lao	12
Sri Lanka	612	Libya	77	Mauritania	11
Morocco	598	Croatia	76	Turkmenistan	10
Peru	546	Puerto Rico	75	Montenegro	10
Israel	516	Albania	75	Dominica	8
Kazakhstan	505	Macedonia	73	Cayman Islands	7
Kenya	491	Belarus	72	Antigue and Barbuda	7
Ukraine	487	Bahrain	72	South Sudan	7
Poland	482	Slovakia	70	Bermuda	7
Greece	474	Uruguay	68	Burkina Faso	7
Myanmar	467	Latvia	66	Togo	6
Argentina	440	Paraguay	65	Mali	6
Chile	439	El Salvador	65	Niger	6
Switzerland	405	Botswana	61	Eritrea	5
Guyana	405	Cyprus	58	Cape Verde	5
Azerbaijan	370	Palestinian Territory	56	Tonga	5
New Zealand	365	Afghanistan	55	Saint Kitts and Nevis	5
Romania	339	Saint Lucia	53	Jersey	5
Ireland	337	Rwanda	52	Benin	5
Lebanon	328	Barbados	51	Turks and Caicos Islands	5
Ecuador	315	Nicaragua	50	Vanuatu	5
Sweden	301	Cameroon	50	Burundi	5
Algeria	298	Sierra Leone	50	Aruba	4
Iraq	292	Zimbabwe	49	Sint Maarten	
Belgium	277	Haiti	46	Seychelles	

spaces (in the sense of Cousin, 2006) leading to satisfying experiences and mastery of key *threshold concepts* (in the sense of Meyer and Land, 2003) in foundational mathematics.

2. Background and motivation

The discovery of calculus independently by Newton and Leibniz, in the Seventeenth Century, was one of the most profound and influential human intellectual achievements of all time, setting off chain reactions of scientific progress and developments that continue to accelerate (Sandlands, 2013). Calculus and its ramifications form the backbone of almost all applications of mathematics to physical and biological sciences and engineering (see, for example, Stewart, 2016; Stewart & Day, 2016; Kreyszig *et al.*, 2011; Spivak, 2019; and Courant & Robbins, 1996).

The MOOC explores and develops powerful themes and narratives, emphasising connections and historical contexts or underpinnings, giving participants opportunities to learn in the higher relational and extended abstract phases of the SOLO taxonomy (Biggs & Collis, 1982). The logo (Figure 1) encapsulates the unity and depth of connections as participants move from precalculus (theory of functions) into differential calculus (theory of derivatives). It depicts the graphs of the natural logarithm and exponential functions, which are mutually inverse, obtained from each other by reflection in the diagonal. These have the crucial properties that the tangent lines to both curves at the intercepts with the axes have slope one. These imply that the exponential function to the base e reproduces itself exactly under differentiation, and the derivative of the natural logarithm is the reciprocal function.

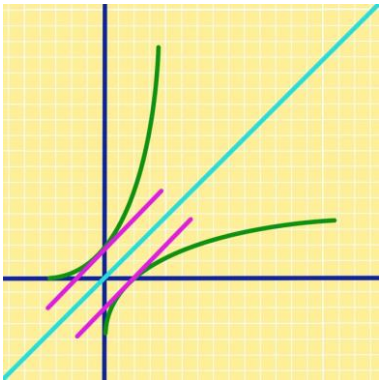

Figure 1. MOOC logo. Source: Timothy Harland.

There is an overwhelming abundance of material on the internet related to elementary calculus and precalculus, so one may ask whether this MOOC might become a superfluous addition to an already cluttered milieu. In fact, the volume of existing material on the internet may be overwhelming to a novice or naïve student, who is unable to tell in advance what

material could be of good or high quality, and then know how to effectively navigate through it. Too much choice can lead to paralysis, and poor choices in navigating the internet can trip people up and even exacerbate phobias or learning difficulties in mathematics.

The MOOC *Introduction to Calculus* however provides an integrated and concise course that encourages depth of learning and guides students to see and experience connections. It looks at calculus from standpoints that may depart considerably from what pupils typically see at school or read in textbooks. A central theme is the way areas and perimeters of regions in the plane are connected by the derivative (in fact the central idea behind the Fundamental Theorem of Calculus), and, stepping up a dimension, also between volumes and surface areas of regions in space. A subtle example discussed in the MOOC, that explores interplay between finite and infinite mathematics, is the melting ice block problem (see Figure 2), predicting how long it takes for an ice block to disappear completely. The rate of melting is proportional to surface area, and the mathematics determines that the width of the ice block diminishes at a constant rate, so indeed it really does disappear after a finite amount of time. If the rate of melting had been proportional to the volume of the cube (analogous to decaying radioactive material) then the mathematics would produce an exponential decay model so that the ice block, though becoming vanishingly small, would never completely disappear (shades of Zeno's Paradox!).

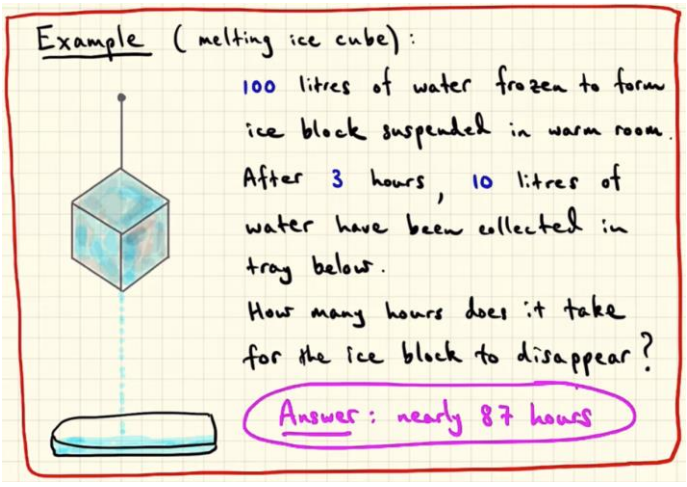

Figure 2. Melting ice cube problem. Source: MOOC Introduction to Calculus.

3. Structure and content

The 'five-week' course repeats indefinitely and is divided into five modules, each of which corresponds nominally to a 'week' of participation and study. These fixed features of the MOOC are glued together by interactive Discussion Forums that enable students to discuss

problems and issues with instructors and other students and to create new threads or respond to existing threads. Participation in the Discussion Forums greatly enhances the learning experience as participants wrestle with and articulate ideas and stumbling blocks in an interactive collegial online environment. Students taking the MOOC

- gain familiarity with key ideas of precalculus, including the manipulation of equations and elementary functions (‘first two weeks’);
- develop fluency with the preliminary methodology of tangents and limits, and the definition of a derivative (‘third week’);
- develop methods of differential calculus with applications (‘fourth week’);
- develop methods of the integral calculus, and see the underlying reasoning behind the Fundamental Theorem of Calculus and original thinking and philosophy of Newton and Leibniz as they explored the origins of calculus (‘fifth week’).

For each module, there is one introductory video and cascades of triplets of materials,

- where each triplet typically comprises one video (sometimes split into two parts), followed by one set of notes, followed by one formative quiz,

finishing with one final summative quiz. The MOOC uses a *mastery learning* model (Bloom, 1968), with the intention that participants use thorough practice and review to move carefully through the *unistructural* and *multistructural phases of learning* in SOLO (Biggs & Collis, 1982), to finally spend time exploring ideas, theories and novel applications in the *relational* and *extended abstract phases*. All multiple-choice quizzes require seventy per cent of answers to be correct, to be regarded as demonstrating a satisfactory passing performance. Students need to pass summative quizzes to progress through the modules, and at the end of the last (fifth) module to be regarded as having completed the MOOC. Multiple attempts are allowed, though each summative quiz may not be attempted more than three times in eight hours. Alternatives for each question are randomised for each attempt. The MOOC develops mathematical writing and communication skills, by participation in the Discussion Forums, which permeate the MOOC, of which there are now thousands of archived threads, attracting contributors on a daily basis.

4. Demographics

At time of writing there have been over 145,000 enrolments, of which over 90,000 are active, completing assessment tasks, with over 12,200 completions. Of the active participants, there is a 70% male to 28% female split. There are significant numbers of teenagers taking the MOOC, even as young as 13-17, which relates to the need for improving pathways from secondary to tertiary mathematics. There are significant numbers of older participants, related

to the use made by teachers to upgrade their skills in imparting calculus to their own students, but also to people in retirement. It is important and beneficial to the community and society that awareness of mathematics is valued by people of all ages. The largest proportion of participants reside in North America, though there are significant proportions from Asia, Europe, Africa, South America and Oceania (see Table 1). Trends in daily and monthly active learners, since May 2019, can be viewed from Figure 3, and the peaks and troughs correlate roughly with the huge waves of infections from COVID-19 and its variants, since early 2020, and imperatives for many educational institutions to use remote learning.

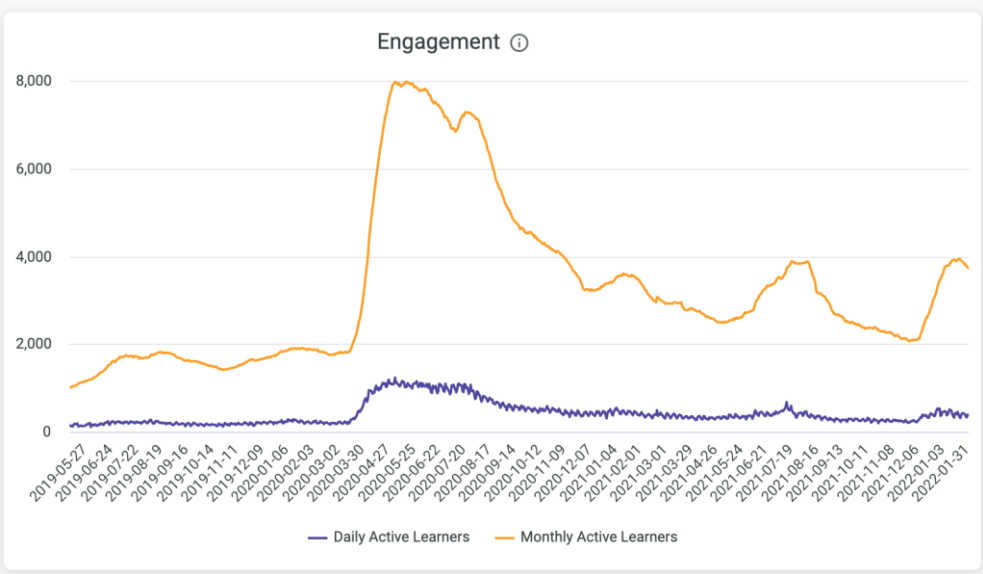

Figure 3. Daily and monthly active learners. Source: Coursera Analytics (1 February 2022).

5. Feedback

At time of writing there have been 2,721 ratings for an average of 4.8/5. Since the launch of the MOOC there have been 837 reviews and 157 learner stories received as well as thousands of comments in the Discussion Forums and through feedback mechanisms related to course development and improvement. The reviews may be read from the following link:

https://www.coursera.org/learn/introduction-to-calculus

Almost all of the comments have been highly positive and constructive. Occasionally there are comments asking for the inclusion of more involved topics or difficult exercises, or for advice or recommendations about continuations in mathematics. It is especially gratifying to hear of teachers who find the materials useful in their own professional development, leading

to improvements in passing on the knowledge and understanding to their own students, and also from students who had previously struggled with mathematics but found the MOOC liberating or revelatory or useful in their academic pathways and professions.

6. Conclusion

The MOOC *Introduction to* Calculus, launched in December 2018, fulfilled an immediate and urgent imperative for an alternative pathway for students who had not studied calculus at school but needed appropriate mathematics prerequisites to enrol in courses at Sydney University from 2019. The MOOC however had been conceived and constructed more broadly and holistically, and emphasises important themes and ideas in mathematics and their historical context and underpinnings. It therefore appeals to a very wide audience, including teachers and other professionals needing to upgrade their skills, and people of all ages throughout the world, who may be simply curious about mathematics and one of the greatest intellectual feats of humanity, the creation of calculus.

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