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"Building" knowledge by creating manipulatives with the 3D printer: a course for mathematics student teachers

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Abstract

3D modelling and printing (3DMP) is a suitable context for education. With it, own manipulatives for mathematics education can be created, which was not possible easily before. We develop a seminar at Goethe University, in which use 3DMP to teach mathematics student teachers about mathematics, pedagogy, and technology. In this paper, we present the general 3DMP process, and how we integrate it in the seminar. The structure and contents of the seminar, as well as results from the students are laid out. The hardware and software used are shown and explained, as well as our approach to 3D modelling with the help of programming. The individual iterations of the seminar and the conclusions show that students mainly benefit from being able to realize their ideas very early and having the freedom to choose their own topic.

Keywords: *3D modelling and printing; mathematics education; project-based learning.*

1. Introduction: 3D Printing

With 3D printing it has become possible to create anything that fits on or in a printer. There are some variations on how 3D printers work: some work by using powder and glue, while others use lasers to harden special resins. The method mostly used in affordable printers is additive manufacturing (FDM, “Fused Deposition Modeling”), which will be referred to in the following as 3D printing (3DP). In 3DP, the printer heats a special form of plastic (“filament”) until it reaches the right temperature (and therefore viscosity), and then “prints” it on top of a print bed—or the previous layers (see figure 1).

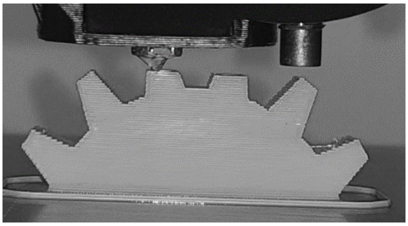

Figure 1: The layers of an additive 3D-print. The upper layers are printed on top of the lower layers.

In general, 3D modelling and printing (3DMP, see Anđić et al., 2022, Läufer & Ludwig, in press) follows the following steps (also shown in figure 2), with an example of a mathematical object (Menger sponge with recursion depth of 2) in figure 3:

1. The process starts with a concept or aim that is to be achieved and notes and calculations are written down.
2. Inside a modelling program or application, a 3D model is created with the help of said notes (Figure 3, top left).
3. This 3D model is then imported into a so-called “slicing”-Software, which “slices” the model into layers (figure 3, top right) and creates a file with a series of commands specific for each printer model.
4. The printer then prints the model in layers by following the commands inside the file (Figure 3, bottom left).

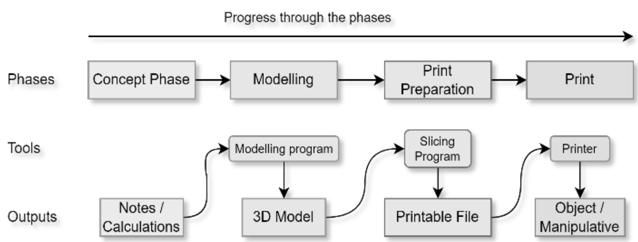

Figure 2: The 3D Modelling and Printing (3DMP) process, divided into phases, tools, and outputs. From every step, we can go towards an earlier step in case a change is needed. After the process has been completed, the manipulative can be validated if it fulfills the aim by using it in class.

This process is iterative. After a first prototype has been printed, multiple changes might be done in following iterations. Either expanding or modifying the original concept (by shifting focus), model, or commands for the printer in the file. This does not only happen after a process has been finished. It may also occur in previous phases. For example, in the slicing program, we can see whether a 3D model may be printable or not, sometimes requiring a remodelling in the modelling program before printing.

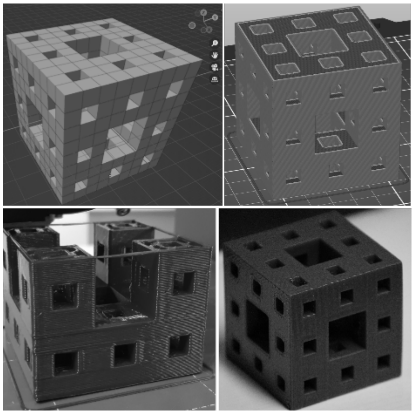

Figure 3: The Menger sponge in the modelling program (top left), the slicing program (PrusaSlicer, top right), during the print (Prusa I3 Mk3S+, bottom left) and printed (bottom right).

In their meta-study, Ford and Minshall (2019) have compiled multiple educational uses of 3D printers. Teachers using 3D printers in their lessons is popular, which has also shown in the studies by Pearson and Dubé (2022) and Kit Ng, Tsui, and Yuen (2022). Ford and Minshall (2019) also report that 3DP in school brings a range of benefits, namely increased motivation and realism inside the classroom, raised student engagement, interest in subject material, and student understanding in the respective fields. Pearson and Dubé (2022) identified multiple studies showing promising effects for (mathematics-) education, such as increased mental rotations skills and understanding through visualization. (Huang & Lin, 2017; Katsioloudis & Jones, 2015) However, these effects are only possible with significant effort of the teacher. (Ford & Minshall, 2019)

While this opens possibilities for students, teachers, and educators, the negatives of 3D(M)P are considerable: long print times for even small objects, a large technical overhead, and a high need for instructor support and training indicate that the complexity of 3D(M)P may not be fully advantageous and integration of 3DMP into lessons faces difficulties. Educators find it difficult to change from their traditional methodology towards a new one (Medina Herrera, Castro Pérez, & Juárez Ordóñez, 2019), and the professional development opportunities needed to use 3DMP in school as a teacher are missing in current teacher education. (Ford & Minshall, 2019; Simpson, Williams, & Hripko, 2017)

2. The Seminar

As described above, 3DP is a rich possibility to produce even complex models according to one’s own ideas. Therefore, we offer a 3DP seminar at Goethe-University, in which student teachers can develop and print suitable manipulatives for their teaching ideas in the context of mathematics education. The seminar (see figure 4) has been developed for the education of 10-12 mathematics student teachers (at least in the 3rd semester). It is the aim to teach them the entire process of 3DMP in one semester. In particular, the seminar aims to increase the mathematical, technical, and pedagogical knowledge and to foster the creativity of the students. The first iteration of the seminar (in the following: “first iteration”) was held from April-July 2022 (13 weeks). The second iteration of the seminar (in the following: “second iteration”) was held from October 2022 through February 2023 with a span of 15 weeks. Between the sessions (2x 45 min/week) the students could make appointments to print their models with the university's printers under the condition of them being present for the entire printing time.

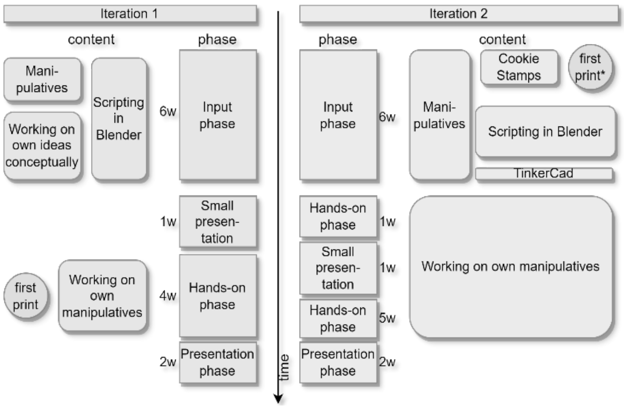

Figure 4: The structure, basic contents, and duration of each phase of the seminar per iteration. On the left side are the elements of the first iteration. The right side holds the second iteration. The “first print” of the first iteration (left) was the first time the students completed the 3DMP process themselves. In the second iteration, the “first print” was in the second session.

The student teachers must create and present manipulatives (individually) that are unique, not easily manufactured by other means, available in shops for a reasonable price, embedded in a lesson plan, complex to a certain degree (i.e., a single simple irregular shape is not sufficient), and do not exist or do only exist virtually, like a sketch or drawing in a book.

2.1. Software use, hardware used and script-based modelling

3DMP is a technologically advanced topic. At least 3 of the 4 phases of the 3DPM process (3D modelling, Print Preparation, and Print, see figure 2) use specific technology and applications. Common 3D modelling applications used for 3DMP may be TinkerCad, Fusion360, SketchUp, and many more. All of them are complex programs, which try to find the balance between usability and powerfulness of the modelling possibilities. We set the criterion that the students must use their mathematical skills to think about all aspects, geometric objects used for, and dimensions of their manipulative before starting the modelling.

Therefore, we chose the free, open-source modelling program Blender. Blender meets the set criterion since it supplies an integrated interface to use scripts – small programming snippets – for 3D modelling. We then developed a library of programmatic functions that reduce the complexity of Blenders' internal interface. This allows script-based modelling of 3D models with a low-threshold programming language, with us being able to adjust the complexity of the programmatic functions and therefore scaffold the students receive. For example, as shown in figure 5, the function "box" only needs two points to create a cuboid. It also supplies the benefit of being precise and the students needing to be prepared before they start modelling, requiring them to think before they do. Depending on the object, demanding calculations are required by the students during 3DMP (Kostakis, Niaros, & Giotitsas, 2015; Nemorin & Selwyn, 2017, cited by Pearson & Dubé, 2022). Depending on the complexity of the manipulative, this may be different from other modelling applications where objects can be dragged, dropped, and edited. A precise measurement may only be needed for complex objects. Additionally, algorithmic structures like loops can be used in the scripts as well, allowing manipulatives like the hundreds chart in figure 5 to be created efficiently and precisely, without needing to place 100 individual cuboids *manually*.

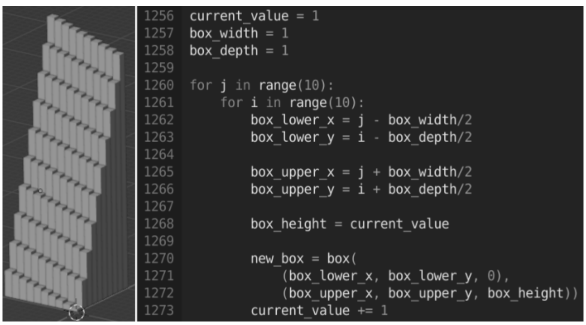

Figure 5: The hundreds field (left, zoomed out to fit) and the code that created it (right) in the Blender scripting area. There are two nested loops in lines 1260 and 1261, each iterating the respective variables – representing the row j and the column i – from 1 to 10. The programmatic function "box()" in lines 1270-1273 creates a cuboid where one side is parallel to the x-y plane, and where the given points (line 1271 is one (x,y,z) point, line 1272 is the other) are in the corners diagonally away from each other.

Slicing software takes the 3D model and creates a set of instructions for a specific type of printer, due to different types and brands having different components and calibrations. The slicing software, PrusaSlicer, is not only free, but tailored to our printers by the creators themselves, reducing the need to fine-adjust the settings in the slicer to get good prints.

The printers we use (Prusa I3 Mk3S/S+) are well suited for shared use, like in the university or school, due to the features the printer has, like a detachable print bed, auto-calibration of the print head, and a filament sensor, which makes switching filaments easy. We chose these printers because they are reliable and there is a slicing software tailored to them.

2.2. First Iteration

The first iteration started with a presentation of the basics of manipulatives and the basics of the 3DMP process. By showing an example of how to create a 3D puzzle, we presented each step. Afterward, we introduced the students to the scripting library, and they started conceptualizing their first ideas, which were then presented shortly to the group in the 5th week. Feedback on the progress was given. This phase was called *input*-phase.

The first conceptualizations showed that the students did not understand the printers’ limits or misunderstood the justification for 3DP to create manipulatives. Some designed long, floating pieces that cannot be printed without significant changes in their design, while others used the 3D printer just for the reproduction of pre-existing manipulatives that can be bought cheaply or easily created with alternative materials. We traced this back to us not enabling the students to print their ideas in the beginning. The first time that students printed something of their design was after 7 weeks (see Figure 4). After the feedback, we continued giving input on the phases and then began the *work*-phase, where attendance was not needed. These could be used for working on the concepts, printing, and asking questions. This phase is called *hands-on*-phase. The objects that the students then created and printed were (among others):

- A puzzle for the cubic formulae with different levels of complexity and distractors,
- A puzzle on approaching the surface of a sphere with triangles,
- A board and pieces for visualizing a path in statistics as a game.

2.3. Changes in the second iteration

Learning from the first iteration, we began the second iteration similarly, but let the students create and print their own mathematically themed cookie stamps in the beginning. Due to the small size and limited complexity, these could be printed in about 20–30 minutes. With our 3 available printers, we managed to print them for almost all students. The only part in the 3DMP we took over was the slicing of the model due to the amount of experience required.

After the students experienced the complete 3DMP process themselves and the input sessions on how to create their own, the students could focus more on the mathematical and didactical side of the process. In the following sessions, more complex models were created by the student teachers with scripts. There were more (creative) and realistic ideas than in the previous iteration, and the students hesitated significantly less to start printing their ideas.

We found that allowing the students to print their models (cookie stamps) early on, was truly motivating. This motivation was upheld by the students, with work-sessions (no attendance required) being filled with almost all students. The evaluation of the seminar was also very positive. Some examples of what students are developing are:

- Platonic Solids as a solid object and their dual solid as an edge model.
- A hundreds-field for early and special education, where the value of the numbers can be felt. (See figure 5 for the 3D model)
- A puzzle of the partner numbers to ten.

We also used one session, right before the *hands-on*-phase, to present a different way to create 3D models with TinkerCad, which has been used by some students to create parts of other models. This was done to allow the students to create simple prototypes more quickly and test the printer settings (like different layer heights) more easily.

3. Conclusion and what is to come

In this paper, we have presented how 3DMP works, as well as how we used it in the context of a seminar at Goethe University. The comparison of iteration one and two makes clear that showing the whole process early on is crucial for student motivation and creativity. The students used a lot of their own ideas and can handle the 3D modelling, preparing the print, and printing on their own. Some support, especially on the technical side, needs to be given nonetheless. The students in the second iteration were motivated and produced innovative manipulatives for mathematics education. Further integration and studies thereof in other subjects such as computer science are of high interest.

The following iterations will embrace the work more with project diaries that will be used as a basis for a PhD-research. Additional interviews with the students on how the design and print processes work for them individually will be held and analyzed. We believe that letting students print their own manipulatives increases their mathematical, didactic and technical knowledge (see TPACK Model, Mishra & Koehler, 2006). The individual 3DMP phases which the students experience and their relation to the TPACK model are to be investigated in this research project.

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